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ASYMPTOTIC METHODS IN DYNAMIC CONTACT PROBLEMS FOR AN ELASTIC HALF-SPACE

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The following dynamic contact problems of the theory of elasticity are considered: (1) the problem of antiplane shear of an elastic half-space by a punch and (2) the plane problem of pressing of a punch into an elastic half-plane. We assume that at time $t = 0$, a force that varies arbitrarily in time is applied to the punch.

To solve these dynamic problems, we apply the Laplace–Carson transform with respect to time and the Fourier transform with respect to the spatial coordinate. As a result, the Laplace–Carson-transformed contact stress problems are reduced to Fredholm integral equations of the first kind of the convolution type on a finite interval, with kernels depending on the dimensionless parameter $\lambda \in (0, \infty)$ related to time.

To solve these equations, we used the methods of [1, 2]. For large and small values of λ, which correspond to large and small times of interaction of a punch and a half-space, simple analytical solutions are obtained in several forms, each of which is effective in its region of variation of the parameter λ. Calculations have shown that these regions overlap the entire possible range of variation of λ. To obtain the final solution of the problems, in the resulting formulas, we go over from the Laplace–Carson transform of unknown functions to their originals.

1. Let an isotropic elastic half-space be subjected to pure shear under the effect of an infinite undeformable band of width $2a$ loaded along its generatrix by a shearing force $T(t) = T_0 f(t)$ [$f(t)$ is a bounded function with a finite number of discontinuity lines for $t \geqslant 0$] related to a unit length. We assume ideal contact between the surfaces of the band and the half-space. We choose an orthogonal coordinate system $Oxyz$. The contact plane $y = 0$ coincides with the interface between the band and the half-space; the half-space occupies the region $y \leqslant 0$. The z axis is directed along the band's generatrix.

The problem is reduced to the solution of the differential equation

$$\Delta w = \frac{1}{c_2^2} \frac{\partial^2 w}{\partial t^2} \quad \left(c_2^2 = \frac{G}{\rho}, \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right), \tag{1.1}$$

which results from the Lamé equations in the absence of mass forces under the boundary conditions (1.2)

$$y = 0: \quad w = \gamma f(t) \quad (|x| \leqslant a), \quad \tau_{yz} = 0 \quad (|x| > a) \tag{1.2}$$

and the initial conditions

$$t = 0: \quad w = 0, \quad \partial w / \partial t = 0. \tag{1.3}$$

Here $w(x, y, t)$ is the projection of the displacement vector onto the z axis, ρ and G are the density and shear modulus of the material of the elastic half-space, τ_{yz} is the tangential component of the stress tensor, and $\gamma f(t)$ is a function that characterizes the rigid displacement of the band.

We shall solve the mixed boundary-value problem (1.1)–(1.3) using integral transforms [3]. Applying

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the Laplace–Carson transform with respect to time

$$w^L = p\int_0^\infty w(x,y,t)e^{-pt}dt, \quad w = \frac{1}{2\pi i}\int_L w^L(x,y,p)\frac{e^{pt}}{p}\,dp, \tag{1.4}$$

we obtain the following boundary-value problem for the function $w^L(x,y,p)$:

$$\Delta w^L = p^2c_2^{-2}w^L, \quad y=0: \quad w^L = \gamma f^L(p) \quad (|x|\leqslant a), \quad \tau_{yz}^L = 0 \quad (|x|>a). \tag{1.5}$$

To solve (1.5), we use the integral Fourier transform with respect to x [1] and write an integral equation with respect to the Laplace–Carson-transformed contact shear stresses $\tau^L(x,p)$. Using the dimensionless variables $x = x'a$ and $\xi = \xi'a$ and the notation $\gamma = \gamma'a$, $\lambda = c_2(ap)^{-1}$, and $\varphi^L(x',p) = \tau^L(x,p)G^{-1}$ (the prime is further omitted), we write this equation as

$$\int_{-1}^{1}\varphi^L(\xi,p)k\Big(\frac{\xi-x}{\lambda}\Big)\,d\xi = \pi g(x)f^L(p) \quad (|x|\leqslant 1); \tag{1.6}$$

$$k(s) = \int_0^\infty K(u)\cos(us)\,du = K_0(s), \quad K(u) = \frac{1}{\sqrt{1+u^2}}, \quad g(x)\equiv\gamma, \tag{1.7}$$

where $K_0(s)$ is the MacDonald function.

We consider the second contact problem on frictionless pressing of a rigid punch of width $2a$ in the elastic half-plane $|x|<\infty$, $y\leqslant 0$ by the force $P(t) = P_0f(t)$, i.e., it is required to find a solution of the Lamé system of equations, which is written for convenience in terms of the wave functions $\varphi(x,y,t)$ and $\psi(x,y,t)$ as

$$\begin{gathered}\frac{\partial}{\partial x}\Big(c_1^2\Delta\varphi - \frac{\partial^2\varphi}{\partial t^2}\Big) + \frac{\partial}{\partial y}\Big(c_2^2\Delta\psi - \frac{\partial^2\psi}{\partial t^2}\Big) = 0,\\ \frac{\partial}{\partial y}\Big(c_1^2\Delta\varphi - \frac{\partial^2\varphi}{\partial t^2}\Big) - \frac{\partial}{\partial x}\Big(c_2^2\Delta\psi - \frac{\partial^2\psi}{\partial t^2}\Big) = 0, \quad c_1^2 = 2G(1-\nu)[\rho(1-2\nu)]^{-1}\end{gathered} \tag{1.8}$$

and is subject to the boundary and initial conditions

$$\begin{gathered}y=0: \quad 2\frac{\partial^2\varphi}{\partial x\partial y} + \frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} = 0 \quad (|x|<\infty),\\ \frac{1}{b^2}\frac{\partial^2\varphi}{\partial y^2} + \Big(\frac{1}{b^2}-2\Big)\frac{\partial^2\varphi}{\partial x^2} + 2\frac{\partial^2\psi}{\partial x\partial y} = 0 \quad \Big(b = \frac{c_2}{c_1}, \quad |x|>a\Big),\\ \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x} = -[\gamma - r(x)]f(t) \quad (|x|\leqslant a);\end{gathered} \tag{1.9}$$

$$t=0: \quad \varphi=\psi=0, \quad \frac{\partial\varphi}{\partial t} = \frac{\partial\psi}{\partial t} = 0. \tag{1.10}$$

In (1.8) and (1.9), ν is the Poisson ratio for the material of the elastic half-plane, $\gamma f(t)$ is the rigid displacement of the punch under the action of the force $P(t)$, and $r(x)$ is a function that describes the punch-base shape. Here we confine ourselves to the case in which the force $P(t)$ is applied at the center of the punch collinearly to the y axis.

To (1.8)–(1.10) we apply in sequence the Laplace–Carson integral transform with respect to time and the Fourier transform with respect to the x coordinate. Thus, we reduce, as before, the solution of the mixed boundary-value problem to the equivalent integral equation in the Laplace–Carson images. This equation in the above dimensionless variables takes the form (1.6). The kernel of the equation is representable in the form (1.7), where

$$K(u) = \frac{2(1-b^2)\sqrt{u^2+b^2}}{(2u^2+1)^2 - 4u^2\sqrt{u^2+b^2}\,\sqrt{u^2+1}} \quad (b^2<1). \tag{1.11}$$

In (1.6), $\varphi^L(x',p) = q^L(x,p)[2G(1-b^2)]^{-1}$, $g(x') = a^{-1}[\gamma - r(x)]$, and $q^L(x,p)$ is the Laplace–Carson transformant of contact pressures (dimensionless variables are unprimed).

2. Before solving the integral equation (1.6), we study the properties of its kernel. It should be noted that the function $K(\zeta)$ (the kernel symbol) is real, even, and positive throughout the real axis and is multivalued in the complex plane $\zeta = u + iv$. In the first of the problems, this function has one pair of branching points $\zeta = \pm i$, while in the second it has two pairs of branching points ($\zeta = \pm i$ and $\zeta = \pm bi$) and two poles [$\zeta = \pm Ri$ $(R > 1)$] located on the imaginary axis. To make the function $K(\zeta)$ single-valued, in the complex plane we make cuts from point $+i$ to point $-i$ through a point at infinity for the first problem, and for the second problem we make two cuts connecting the branching points bi with i and $-bi$ with $-i$.

The asymptotic properties of the function $K(\zeta)$ are as follows:

$$K(u) = A + O(u^2) \quad (u \to 0), \quad uK(u) = 1 + \sum_{n=1}^{N} e_{2n} u^{-2n} + O(u^{-2N-2}) \quad (u \to \infty). \tag{2.1}$$

Here $A = 1$ for the first problem and $A = 2b(1-b^2)$ for the second problem.

It follows from (2.1) that, for small s, the asymptotics of the kernel $k(s)$ is of the form [2]

$$k(s) = \ln|s| l_1(s) + l_2(s), \quad l_j(s) = \sum_{i=0}^{N} d_{ji} s^{2i} + O(s^{2N+2}) \quad (j = 1,\ 2), \tag{2.2}$$

where the first several coefficients d_{ji} are given by the formulas

$$d_{10} = -1, \quad d_{11} = \frac{e_2}{2}, \quad d_{12} = -\frac{e_4}{24}, \quad d_{20} = \int_0^\infty \Big[K(u) - \frac{1 - e^{-u}}{u}\Big]\, du,$$

$$d_{21} = -\frac{3}{4} e_2 + \frac{1}{2} \int_0^\infty [u^2 - u^3 K(u) + e_2(1 - e^{-u})] \frac{du}{u},$$

$$d_{22} = \frac{25}{288} e_4 - \frac{1}{24} \int_0^\infty \Big[u^4\Big(1 - \frac{e_2}{u^2}\Big) - u^5 K(u) + e_4(1 - e^{-u})\Big] \frac{du}{u}.$$

We seek a solution of the integral equation (1.6) for large λ in the form [2]

$$\varphi^L(x,p) = \sum_{n=0}^{N} \sum_{m=0}^{n} \varphi^L_{2n,m}(x,p) \lambda^{-2n} (\ln\lambda)^m + O(\lambda^{-2N-2} \ln^{N+1}\lambda). \tag{2.3}$$

Substituting (2.2) and (2.3) in (1.6), equating expressions of equal powers of λ^{-2} and $\ln\lambda$, and taking the inverse Laplace–Carson transform of $\varphi^L(x,p)$, we obtain with accuracy up to terms of the order of $O(\lambda^{-4}\ln\lambda)$

$$\varphi(x,t) = \varphi_{00}(x,t) + \frac{1}{\Lambda^2} \frac{\partial^2}{\partial t^2} \varphi_{20}(x,t) + \frac{1}{\Lambda^2} \frac{\partial}{\partial t} \int_0^t \frac{\varphi_{21}(x,\tau)}{(t-\tau)^2}\, d\tau$$

$$- \frac{\mathrm{C}}{\Lambda^2} \frac{\partial^2}{\partial t^2} \varphi_{21}(x,t) - \frac{\ln\Lambda}{\Lambda^2} \frac{\partial^2}{\partial t^2} \varphi_{21}(x,t) \quad (\Lambda = \lambda p), \tag{2.4}$$

$$p^2(\ln p + \mathrm{C}) \risingdotseq \frac{1}{t^2}, \quad p \risingdotseq \frac{\partial}{\partial t}, \quad f^L(p) g^L(p) \risingdotseq \frac{d}{dt} \int_{-0}^{t+0} g(\tau) f(t-\tau)\, d\tau;$$

$$\varphi_{00}(x,t) = \frac{1}{\pi\sqrt{1-x^2}} \Big[N_0(t) - f(t) \int_{-1}^{1} \frac{\sqrt{1-\xi^2}}{\xi - x} g'(\xi)\, d\xi\Big],$$

$$\varphi_{20}(x,t) = \frac{1}{\pi^2\sqrt{1-x^2}} \int_{-1}^{1} \frac{\sqrt{1-\xi^2}}{\xi - x} d\xi \int_{-1}^{1} (\xi - \tau)(2d_{11} \ln|\xi - \tau| + 2d_{21} + d_{11})\varphi_{00}(\tau,t)(1-\tau^2)^{-1/2} d\tau, \qquad (2.5)$$

$$\varphi_{21}(x,t) = -\frac{2d_{11}}{\pi^2\sqrt{1-x^2}} \int_{-1}^{1} \frac{\sqrt{1-\xi^2}}{\xi - x}\, d\xi \int_{-1}^{1} \varphi_{00}(\tau)(\xi - \tau)\, d\tau;$$

$$N_0(t) = \int_{-1}^{1} \varphi(x,t)\, dx \qquad (2.6)$$

(C is the Euler constant). Here, the integrals and derivatives in (2.4) should be understood in the generalized sense [3].

The following fact is noteworthy. Let, for example, the punch have a flat base [$g(x) \equiv g = \text{const}$], and $f(t) \equiv H(t)$, where $H(t)$ is the Heaviside step function [3]. Mechanically, this means that the punch has instantaneously moved for the quantity g under the action of the force $N_0(t)$, and it is sustained by the force in this state. Then for large t, formulas (2.4) and (2.5) become

$$\varphi(x,t) = \frac{N_0}{\pi\sqrt{1-x^2}}, \qquad (2.7)$$

where $N_0 = T_0/(aG)$ for problem 1 and $N_0 = P_0/[2aG(1-b^2)]$ for problem 2, i.e., they degenerate rather rapidly into the solution of the corresponding static problem. As is known from [4], for this problem, it is impossible to find a relationship between the force N_0 and the rigid displacement of the punch g from condition (2.6).

We construct a solution of the integral equation (1.6) for small λ (small time). For this, we use the asymptotic method of "small" λ [1, 2]. The zeroth term of the asymptotics of the solution of Eq. (1.6) for $\lambda \to 0$ is representable as

$$\varphi^L(x,p) = \varphi_*^L\Big(\frac{1+x}{\lambda}, p\Big) + \varphi_*^L\Big(\frac{1-x}{\lambda}, p\Big) - v^L\Big(\frac{x}{\lambda}, p\Big), \qquad (2.8)$$

where $\varphi_*^L(y,p)$ and $v^L(y,p)$ are solutions of the integral equations of the form

$$\int_0^\infty \varphi_*^L(\tau,p)k(\tau - y)\, d\tau = \pi g \lambda^{-1} f^L(p) \quad (0 \leqslant y < \infty); \qquad (2.9)$$

$$\int_{-\infty}^{\infty} v^L(\tau,p)k(\tau - y)\, d\tau = \pi g \lambda^{-1} f^L(p) \quad (|y| < \infty). \qquad (2.10)$$

Here we use the evenness of the function $K(u)$ ($|u| < \infty$) and Theorem 24.4 from [1], according which it will suffice to study only Eq. (1.6) with the right-hand side $g(x) \equiv g = \text{const}$.

The solution of (2.9) can be found by the Wiener–Hopf method [1] if the factorization of the function $K(\alpha) = K_+(\alpha)K_-(\alpha)$ is used:

$$\varphi_*^L(y,p) = -\frac{g f^L(p)}{\lambda K_+(0)} \frac{1}{2\pi i} \int_{-\infty+ic}^{\infty+ic} \frac{e^{-i\alpha y}}{K(\alpha)\alpha}\, d\alpha \quad (c > 0). \qquad (2.11)$$

The integral equation (2.10) is solved by means of the integral Fourier transform and the convolution theorem [1]:

$$v^L(y,p) = g f^L(p)[\lambda K(0)]^{-1}. \qquad (2.12)$$

Applying the inverse Laplace–Carson transform to (2.8) and taking into account that $\lambda = \Lambda p^{-1}$, we write solutions of the formulated problems for $\lambda \to 0$ as

$$\varphi(x,t) = \varphi_*\Big(\frac{1+x}{\Lambda}, t\Big) + \varphi_*\Big(\frac{1-x}{\Lambda}, t\Big) - v\Big(\frac{x}{\Lambda}, t\Big). \tag{2.13}$$

Setting $K(\alpha) = (1+\alpha^2)^{-1/2}$ in (2.11), (2.12), we have

$$\varphi_*^L\Big(\frac{1\pm x}{\lambda}, p\Big) = \frac{gf^L(p)}{\lambda}\Big\{\frac{\exp[-\lambda^{-1}(1\pm x)]}{\sqrt{\pi\lambda^{-1}(1\pm x)}} - \operatorname{erfc}\sqrt{\frac{1\pm x}{\lambda}} + 1\Big\}, \quad v^L\Big(\frac{x}{\lambda}, p\Big) = \frac{gf^L(p)}{\lambda}, \tag{2.14}$$

whence, in accordance with (2.13), we find a solution of the problem of pure shear of an elastic half-space by a punch:

$$\frac{\Lambda\varphi(x,t)}{g} = \begin{cases} f'(t) & [0 < t\Lambda < \sigma_-(x)], \\ f'(t) + \dfrac{1}{\pi\sqrt{\sigma_-(x)}}\dfrac{\partial}{\partial t}\displaystyle\int_{\sigma_-(x)\Lambda^{-1}}^{t} f(t-\tau)\,\frac{\theta_-(\tau,x)}{\tau}d\tau & [\sigma_-(x) < t\Lambda < \sigma_+(x)], \\ f'(t) + \dfrac{1}{\pi\sqrt{\sigma_-(x)}}\dfrac{\partial}{\partial t}\displaystyle\int_{\sigma_-(x)\Lambda^{-1}}^{t} f(t-\tau)\,\frac{\theta_-(\tau,x)}{\tau}d\tau & \\ \quad + \dfrac{1}{\pi\sqrt{\sigma_+(x)}}\dfrac{\partial}{\partial t}\displaystyle\int_{\sigma_+(x)\Lambda^{-1}}^{t} f(t-\tau)\,\frac{\theta_+(\tau,x)}{\tau}d\tau & [t\Lambda > \sigma_+(x)], \end{cases} \tag{2.15}$$

$$\sigma_\pm(x) = 1 \pm |x|, \quad \theta_\pm(x,\tau) = \sqrt{\tau\Lambda - \sigma_\pm(x)}.$$

In going from the Laplace–Carson transforms to originals in relations (2.13) and (2.14), we used the following formulas [5]:

$$p\operatorname{erfc}(\sqrt{\alpha p}) \doteqdot \begin{cases} 0 & (t < \alpha), \\ \dfrac{1}{\pi t}\sqrt{\dfrac{\alpha}{t-\alpha}} & (t > \alpha, \alpha > 0), \end{cases} \qquad \sqrt{p}\,e^{-\alpha p} \doteqdot \begin{cases} 0 & (t < \alpha), \\ \dfrac{1}{\sqrt{\pi(t-\alpha)}} & (t > \alpha). \end{cases}$$

If the law of motion for the punch is given, as above, by the Heaviside function, the solution (2.15) is simplified and becomes

$$\frac{\Lambda\varphi(x,t)}{g} = \begin{cases} \delta(t) & [0 < t\Lambda < \sigma_-(x)], \\ \delta(t) + \dfrac{\theta_-(t,x)}{\pi t\sqrt{\sigma_-(x)}} & [\sigma_-(x) < t\Lambda < \sigma_+(x)], \\ \delta(t) + \dfrac{\theta_-(t,x)}{\pi t\sqrt{\sigma_-(x)}} + \dfrac{\theta_+(x,t)}{\pi t\sqrt{\sigma_+(x)}} & [t\Lambda > \sigma_+(x)], \end{cases} \tag{2.16}$$

where $\delta(t)$ is the Dirac delta-function.

Having obtained expressions (2.15) and (2.16), one can analyze the contact tangential stresses in the first problem. From Eqs. (2.15), it is evident that until the contact-stress wave reaches the "observation" point under the punch, the stress at this point is proportional to the punch velocity $f'(t)$. The contact tangential stress wave arriving at the "observation" point from the nearest punch end has zero stress at the front and propagates at the velocity of a transverse wave in the given medium. In addition, after a certain period of time, the contact-stress wave arrives at this point from the other punch end. In particular, from relations (2.16), which correspond to the case $f(t) \equiv H(t)$, it follows that the contact tangential stresses at each point under the punch are proportional to $\delta(t)$, while for $t > 0$ before the arrival of the transverse wave from the nearest punch end, they are zero.

The distributions of contact tangential stresses $\Phi(x,t) = \Lambda g^{-1}\varphi(x,t)$ are plotted in Fig. 1 for times $t\Lambda = 0.5$, 1.0, and 1.5 (curves 1–3). In addition, Fig. 2 illustrates the quantity $\Phi(x,t)$ versus $t\Lambda$ at the points $x = 0$, 0.5, and 0.9 (curves 1–3).

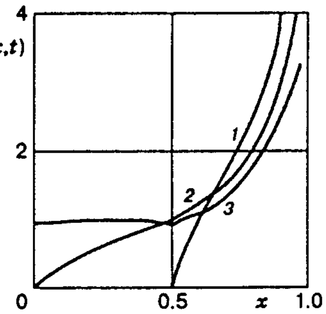

Fig. 1

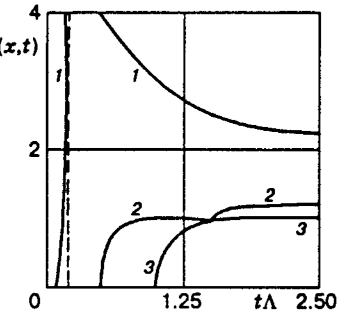

Fig. 2

An important characteristic of solutions of contact problems is the force (2.6) acting on the punch. To find the force, we use the fact that, for small λ, the zeroth term of the asymptotic solution of the integral equation (1.6) is representable in multiplicative form [1]:

$$\varphi^L(x,p) = \varphi_*^L\left(\frac{1+x}{\lambda}, p\right)\varphi_*^L\left(\frac{1-x}{\lambda}, p\right)\left[v^L\left(\frac{x}{\lambda}, p\right)\right]^{-1}, \tag{2.17}$$

where the functions $\varphi_*^L(y,p)$ and $v^L(y,p)$ satisfy Eqs. (2.9) and (2.10). Let us first determine the quantity

$$N_0^L(p) = \int_{-1}^{1} \varphi^L(x,p)\,dx. \tag{2.18}$$

Introducing (2.11), (2.12), and (2.17) in Eq. (2.18) and performing the necessary transformations [1], we obtain

$$N_0^L(p) = gf^L(p)\frac{K_-(0)}{K_+(0)}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{2\alpha/\lambda}}{\alpha^2 K_+^2(i\alpha)}\,d\alpha. \tag{2.19}$$

Assuming that $K(\alpha) = (1+\alpha^2)^{-1/2}$, from (2.19) we obtain

$$N_0^L(p) = gf^L(p)(2\lambda^{-1}+1). \tag{2.20}$$

Applying the inverse Laplace–Carson transform to (2.20), with allowance for the last relation in (2.4) we find

$$N_0(t)g^{-1} = 2\Lambda^{-1}f'(t) + f(t). \tag{2.21}$$

If $f(t) \equiv H(t)$ in Eq. (2.21), then $N_0(t)g^{-1} = 2\Lambda^{-1}\delta(t) + H(t)$.

3. We construct a solution of the second problem for small times. It should be taken into account that in this case, as was mentioned above, the kernel symbol (1.11) of the integral equation (1.6) is of a more complex form than that in the first problem. Moreover, it contains four branching points in the complex plane, which significantly complicates factorization of $K(\zeta)$. Nevertheless, the function $K(\zeta)$ ($\zeta = u + iv$) is positive and does not have singular points such as poles and branching points on the real axis.

Taking into account this fact and also the asymptotic properties of (2.1) and using the Coiter method of approximate factorization [6], we approximate the kernel symbol $K(\zeta)$ (1.11) by the expression[1]

$$K(\zeta) \approx \sqrt{\zeta^2 + h_1^2(\zeta^2+h_2^2)^{-1}}, \quad h_1 = b, \quad h_1 h_2^{-2} = 2b(1-b^2). \tag{3.1}$$

[1]An approximation of the kernel symbol $K(\zeta)$ of the form (1.11) that takes into account its behavior over the entire complex plane $\zeta = u + iv$ is given in [7].

For the function $K(\zeta)$, we can propose a simpler approximation than (3.1):

$$K(\zeta) \approx (\zeta^2 + h_1^2)^{-1/2}, \quad h_1 = b. \tag{3.2}$$

The approximation of the form (3.2) is chosen from the following considerations. First, the branching point of function (3.2) coincides with the first branching point of the kernel symbol $K(\zeta)$ of the form (1.11). Therefore, one can speak about the approximation of $K(\zeta)$ in the region $|v| < h_1 < 1$, $|u| < \infty$ in the plane of the complex variable $\zeta = u + iv$. This makes it possible to obtain a physically correct solution, because the propagation velocity of the longitudinal wave of contact stresses under the punch coincides with c_1 [8]. Second, the error of this approximation along the real axis does not exceed 6%.

Introducing $K(\zeta)$ in the form (3.2) in relations (2.11) and (2.19), we arrive at equalities (2.15) and (2.21), in which one should replace the parameter Λ by the expression $\Lambda = c_1 a^{-1}$. Thus, the character of contact stresses here is the same as in the first problem, and, hence, it can be illustrated by Figs. 1 and 2.

In conclusion, we note that the problem of a semi-infinite punch in [8] was solved in approximately the same manner, and the factorization of the function $K(\zeta)$ (1.11) was taken in the form of the Cauchy integrals. The results of [8] are in qualitative agreement with our results.

Moreover, numerical calculations have shown that the solutions of the problems for small and large times coincide in the region $t = 2\Lambda^{-1}$.

Note that we can construct a solution of the integral equation (1.6) and (1.7) by the method of orthogonal functions if we use the spectral relation [9]

$$-\int_{-1}^{1} \frac{\mathrm{ce}_n(\arccos\xi, -æ)}{\sqrt{1-\xi^2}} K_0\left(\frac{\xi - x}{\lambda}\right) d\xi = \pi \frac{\mathrm{Fek}_n(0, -æ)}{\mathrm{Fek}'_n(0, -æ)} \mathrm{ce}_n(\arccos x, -æ),$$

where $|x| < 1$ and $æ = 1/(4\pi^2)$, and the orthogonality property of the Mathieu periodic functions. Omitting intermediate calculations, we have

$$\varphi^L(x,p) = \frac{2gf^L(p)}{\sqrt{1-x^2}} \sum_{n=0}^{\infty} (-1)^{n+1} A_0^{(2n)} \frac{\mathrm{Fek}'_{2n}(0,-æ)}{\mathrm{Fek}_{2n}(0,-æ)} \mathrm{ce}_{2n}(\arccos x, -æ). \tag{3.3}$$

To obtain now a solution of the main nonstationary problem of pure shear of an elastic half-space, one should take, according to (1.4), the inverse Laplace–Carson transform in (3.3). Taking into account the latter relation in (2.4), we write

$$\begin{gathered} \varphi(x,t) = \frac{g}{\sqrt{1-x^2}} \frac{\partial}{\partial t} \int_0^t f(t-\tau) h(x,\tau)\, d\tau, \\ h(x,t) \doteq 2 \sum_{n=0}^{\infty} (-1)^{n+1} A_0^{(2n)} \frac{\mathrm{Fek}'_{2n}(0,-æ)}{\mathrm{Fek}_{2n}(0,-æ)} \mathrm{ce}_{2n}(\arccos x, -æ). \end{gathered} \tag{3.4}$$

Numerical inversion of the Laplace–Carson transform is performed in the second formula of (3.4), for example, by the Papulis method [10]. Thereafter, one can determine the force $N_0(t)$ from (2.6) and from the first formula of (3.4).

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